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On the Unification of Geodetic Levelling Datums Using Satellite Altimetry

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ABSTRACT

Reasons are adduced for the continuing need for continental networks of high precision geodetic levelling. The serious doubts which currently exist concerning the reliability of such networks, prompts the development of new techniques for independently estimating the height of mean sea level (MSL) at coastal tide gauge sites. Techniques are described for achieving these goals from satellite altimetry.

Numerical results are obtained from the 1977 GEOS-3 altimetry data bank at Goddard Space Flight Center. The potential of the geoid based on the Bermuda calibration of the altimeter and the GM value of $3.986\,004\,7 \times 10^{20} \text{ cm}^3 \text{ s}^{-2}$ is 6,263,682.8 kGal m when computed from a forty percent coverage of the oceans, sampled between parallels 65°S and 65°N during periods representative of the equinoxes closest to 1976.0. This value is subject to revision by up to ± 0.3 kGal m with further orbital improvements.

On this basis, it is estimated that the height of MSL at the Jervis Bay Datum for Australia is $+0.2 \pm 0.4$ m. The discrepancy of zero degree between the gravity anomaly file for central North America and the geoid for 1976.0 can be interpreted as an estimate of the height of MSL at a Galveston Datum of $+0.1$ m. These values are in closer agreement with extrapolated oceanographic estimates of steric anomalies at both sites than indicated by the uncertainties given above. The differences in the estimates of MSL from both methods are in good agreement.

From these results, it can be concluded that all gravity data in AUSGAD 76 and in the Rapp gravity file for central North America refer to the geoid for 1976.0 with uncertainties of 0.1 mGal. The technique also provides an exacting test of the value used for GM in the computations.

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ON THE UNIFICATION OF GEODETIC LEVELLING DATUMS USING SATELLITE ALTIMETRY

1. INTRODUCTION

The geodetic levelling operation is fundamental to precise engineering workings involving the dynamics of flow. The propagation of accidental error in first order geodetic levelling networks is not expected to exceed $\pm 2 \sqrt{D}^{(km)}$ mm where D is the length of the line of levelling. The systematic error accumulation is expected to be about one order of magnitude smaller (Bomford 1962, p. 238). The adjustment of such levelling networks over continental extents should provide data at bench marks (BMM) which are internally consistent to better than ± 20 cm or its equivalent.

However, the comparison of the heights of MSL at tide gauge locations obtained from freely adjusted levelling networks, with those obtained from tidal analysis, indicate discrepancies significantly larger than expected from the internal statistics of the network adjustment, primarily in the north-south direction (e.g., Mather 1974b, pp. 70-71). In addition, the deduced slopes of coastal sea surface topography (SST) disagree with estimates obtained from hydrostatic considerations.

In the latter technique widely used in physical oceanography (e.g., Lisitzin 1974, p 72), isobaric and level surfaces are assumed to coincide at great depth in the oceans ($> 10^3$ m). Changes dT in temperature T , dp_a in atmospheric pressure p_a and dS in the salinity S of sea water (density ρ_w), produce changes dh in the dynamic height of the sea surface in relation to that of a standard column of sea water at temperature T_0 (273°K), pressure p_0 (1 atmosphere) and salinity S_0 (35 parts per thousand), up to the depth of no motion h_0 , according to the relation

$$dh = \frac{1}{g} \left(\int_{p_a}^{p_0} \left(\frac{\partial \alpha}{\partial T} \right) dT dp + \int_{p_a}^{p_0} \left(\frac{\partial \alpha}{\partial S} \right) dS dp - \frac{1}{\rho_w} dp_a \right) \quad (1)$$

dp being the incremental change in pressure and α is the specific volume of sea water.

The assumption of a level of no motion implies a coincidence between isobaric and level surfaces in deep oceans — a condition which is free from assumption only in regions exterior to the Earth's atmosphere. Hydrostatic determinations of dynamic sea surface heights are also subject to short period effects, such as frontal movements — a change of 1 mb in pressure causes a 1 cm change in sea

level height. There is also a tendency for the effect of pressure to be correlated with latitude (e.g., Lisitzin 1974, p 67). It has been shown that the irregular lower boundary of the atmosphere causes changes in the contribution of the atmosphere to the height anomaly ζ' of ± 50 cm (Anderson, et al., 1975). The departures from equivalence between isobaric and level surfaces are expected to be a maximum as continental margins are approached.

Global maps of dynamic SST prepared in this manner are not based on data collected simultaneously. It is recognized that the quasi-stationary contribution may contain only a fraction of the power in the spectrum of SST (e.g., Wilson and Dugan 1977).

The two factors mentioned above make some contribution to the discrepancies between coastal comparisons of sea surface slopes from geodetic and oceanographic methods. A third source of uncertainty is the need for extrapolating values of the SST (ζ_s) from deep oceans to coastal tide gauge sites using the Lagrangian equations (e.g., Mather 1976, p 121)

$$\ddot{x}_1 - f \dot{x}_2 = -g \frac{\partial \zeta_s}{\partial x_1} - \frac{1}{\rho_w} \frac{\partial p_a}{\partial x_1} + F_1$$

and

(2)

$$\ddot{x}_2 + f \dot{x}_1 = -g \frac{\partial \zeta_s}{\partial x_2} - \frac{1}{\rho_w} \frac{\partial p_a}{\partial x_2} + F_2,$$

where (\dot{x}_1, \dot{x}_2) , (\ddot{x}_1, \ddot{x}_2) and (F_1, F_2) are components respectively of surface velocity, acceleration and frictional forces of the ocean along the axes (x_1, x_2) of a local two dimensional Cartesian coordinate system in the local horizon plane with the x_1 axis oriented east and the x_2 axis oriented north, f the Coriolis parameter

$$f = 2\omega \sin \phi, \quad (3)$$

ω being the angular velocity of rotation of the Earth and ϕ the latitude.

The use of this technique assumes that current meter measurements of \dot{x}_1, \dot{x}_2 are available, along with measurement of horizontal atmospheric pressure gradients and data for the evaluation of the frictional forces. Practical calculations are performed by assuming a non-accelerated system (i.e., $\ddot{x}_1, \ddot{x}_2 = 0$). Except in abnormal conditions, $\dot{x}_\alpha < 10^2 \text{ cm s}^{-1}$ and $\partial \zeta_s / \partial x_\alpha = o\{0.2\}$. It follows that F_1, F_2 must be estimated to $\pm 10^{-4} \text{ cm s}^{-2}$ ($\pm 0.1 \text{ mGal}$) if extrapolation errors are to be held below $\pm 1 \text{ cm}$. Physical oceanographers have maintained

that extrapolations of values of ξ_s over distances of up to 300 km from deep oceans to coastal sites, are unlikely to introduce errors of more than ± 10 cm in the result (e.g., Hamon and Greig 1972, p. 7160).

In view of the uncertainties surrounding the estimates of coastal SST from oceanographic considerations and the doubts cast on the validity of geodetic levelling (e.g., Sturges 1974, p. 830), it is necessary that an independent means be established to achieve the following objectives:

- Determination of the height of MSL at each tide gauge linked to a geodetic levelling network, with a precision of at least ± 10 cm in the first instance,
- Definition of the universal datum level surface to which each of these MSL heights is referred, with an equivalent precision.

These objectives are of especial interest to the African region where each nation has its own regional levelling datum, some of which have no direct access to the oceans and hence, the geoid. Satellite altimetry provides, in principle, an efficient means of achieving this objective provided the data is handled with due regard to theoretical niceties. This paper defines the basic relations which can provide the foundation for unifying all the world's levelling datums with a precision equivalent to that in the radial component of altimeter-satellite orbit determination. On present indications, this is likely to be ± 10 cm in the foreseeable future, in regions of adequate tracking.

2. SATELLITE ALTIMETRY DATA IN COASTAL REGIONS

Satellite altimetry data in coastal regions has been acquired by the radar altimeter on board the GEOS-3 spacecraft since 1975. The analysis of data in the Tasman and Coral Seas (Mather, et al., 1977) in continental shelf areas off the east coast of Australia, provided at 2 second time intervals with the altimeter operating in the short pulse mode, indicates the following:

- the sea surface appears to rise relatively steeply over the continental shelf slope; and
- non-oceanic readings and hence, the transition from ocean to land, are clearly recognizable at the ± 1 m level between successive data records.

A steep geoid rise in the region is not unexpected from the nature of the surface gravity field. On these figures, it can be conservatively estimated that satellite altimetry may provide data of quality up to 20 km from the coastline, especially when using the ± 10 cm radar altimeter planned for the SEASAT-A spacecraft, due for launch in mid-1978. This altimeter is expected to have a footprint of 2-12 km (Nagler and McCandless 1975, p. 2).

The basic data is in the form of heights ζ' of the instantaneous sea surface above the adopted reference figure. The sequence of operations to convert such data into values of the heights of MSL at the regional tide gauge site is the following:

- a. Determine the heights ζ_s of the quasi-stationary SST in the adjacent continental shelf areas. This presumes that the geoid has already been defined on the basis of a global analysis of values of ζ' .
- b. Extrapolate the resulting values of ζ_s in the shallow continental shelf ocean to the coastal site using Equation (2) and as outlined in the ensuing discussion.

As ζ_s is not greater than ± 2 m, values of ζ' should be computed from orbits which have a resolution of at least ± 10 cm in the radial component of position. Values of the quasi-stationary height of MSL deduced from satellite altimetry should be the average of at least one year's readings. The oceanographic surveys for current velocities, atmospheric pressure gradients and frictional forces can only be carried out only on a few finite occasions, possibly just once. However, the continuous monitoring of local ground truth during the period of altimetry should provide a basis for accurate extrapolation using Equation (2), in most areas.

3. BASIC RELATIONS

The difficulties likely to be encountered in determining ζ_s from satellite altimeter measurements of ζ' in ocean areas have been described at length in a series of papers (Mather 1974a; Mather, et al., 1976a; Mather 1978). On assuming the data to be of adequate precision (i. e., $\pm 1-5$ cm in ζ' ; $\pm 3-15$ μ Gal in the gravity anomaly $\Delta g'$ through wavelengths of interest), the principal problems to be overcome are the following:

- a. No complete coverage exists globally for either ζ' or $\Delta g'$. The precision of oceanic gravity data is at least an order of magnitude worse than that of land gravity data. The former is probably subject to systematic errors of long wavelength. It has been shown that even a homogeneous gravity field determination like that available for Australia is only adequate for SST determinations with a precision of ± 30 cm (Mather, et al., 1976b).
- b. All data are measured in relation to the sea surface, either instantaneous or MSL, and not the geoid.
- c. Local MSL approximates the geoid to no better than ± 2 m (Mather 1977).

Three equivalent relations exist between measured values of the gravity anomaly $\Delta g'$, the height anomaly ξ' in relation to the higher reference system (Mather 1974a, p. 91) and the sea surface topography ξ_s (Mather, et al., 1976a, p. 34). Two of these relations are of use in determining ξ_s through all wavelengths in excess of the Nyquist limit which is defined by the altimeter footprint (i. e., greater than about 20 km). They are the following:

RELATION 1

$$T'' = (W_0 - U_0) - \gamma(\xi_s + k_0 \xi_t) + \gamma \xi' - V, \quad (4)$$

where T'' is the disturbing potential of the solid Earth and oceans in relation to the higher reference system obtained by incorporating the gravity field model determined from orbital analysis (e. g., GEM 9), with the conventional geodetic reference system (e. g., IAG 1971) defined by the rotating equipotential reference ellipsoid with potential U_0 ; W_0 is the potential of the geoid which requires conceptual definition at the ± 10 cm level (Mather 1977); V is the potential of the Earth's atmosphere at the surface of measurement and γ is for all practical purposes, the global mean value of normal gravity. ξ_t is the height of the ocean tide as perturbed by the Earth tide and k_0 is the oceanic function given at points with latitude ϕ and longitude λ by

$$k_0(\phi, \lambda) = \begin{cases} 0 & \text{if } (\phi, \lambda) \text{ on land} \\ 1 & \text{if } (\phi, \lambda) \text{ is oceanic.} \end{cases} \quad (5)$$

RELATION 2

$$\Delta g_{cp} + \gamma N''_{cp} / \bar{R} - \left((W_0 - U_0) - \gamma \left(\xi_{sp} + \frac{1}{4\pi} \iint M_1(\psi) [\xi_s - \xi_{sp}] d\sigma \right) \right) / \bar{R} = \frac{\gamma}{4\pi \bar{R}} \iint M_1(\psi) [N''_c - N''_{cp}] d\sigma, \quad (6)$$

where Δg_c is the computable part of the pseudo-gravity anomaly $\Delta g''$ on the Brillouin sphere (minimum geocentric sphere containing all the Earth's topography), given by the relation

$$\Delta g_c = \Delta g' + \delta g_n + \delta \Delta g'' + o \{ \int \Delta g_c \}, \quad (7)$$

δg_a being the atmospheric correction to observed gravity (Anderson, et al., 1975, p. 25), $\delta \Delta g''$ the change in $\Delta g''$ between the Earth's surface and the Brillouin sphere of radius \bar{R} , the subscript p referring to values at the point of computation. $d\sigma$ in Equation (6) refers to the element of surface area at an angular distance ψ from the point of computation.

The gravity anomaly $\Delta g'$ on the higher reference system can be related to the conventional gravity anomaly Δg by the relation

$$\Delta g' = \Delta g - \delta \gamma, \quad (8)$$

where $\delta \gamma$ is defined by the coefficients $C'_{\alpha nm}$ of an Earth gravity field model like GEM 9 (e.g., Lerch, et al., 1977) by a relation of the form (Mather 1974a, p. 95)

$$\delta \gamma = \gamma \sum_{n=0}^{n'} (n-1) \sum_{m=0}^n \sum_{\alpha=1}^2 C'_{\alpha nm} S_{\alpha nm} + o \left\{ |\delta \gamma| \right\}, \quad (9)$$

$S_{\alpha nm}$ being surface spherical harmonic functions of degree n and order m , given by

$$S_{1nm} = P_{nm}(\sin \phi) \cos n\lambda; S_{2nm} = P_{nm}(\sin \phi) \sin m\lambda, \quad (10)$$

n' being approximately 20.

The quantity N_c'' is related to the height anomaly ζ' - i.e., the height of the sea surface above the reference surface in ocean areas - by the relation (Mather, et al., 1976a, p. 29)

$$N_c'' = \zeta' - \frac{1}{\gamma} (V - \delta T''), \quad (11)$$

$\delta T''$ being the change in T'' between the Earth's surface and the Brillouin sphere. The surface integrals in Equation (6) apply on the Brillouin sphere which is the smallest sphere on which the orthogonal properties of surface spherical harmonics apply without approximation, to the potential T'' and the pseudo-gravity anomaly $\Delta g''$.

The kernel $M_1(\psi)$ of the surface integrals in Equation (6) is given by

$$M_1(\psi) = \sum_{n=2}^{\infty} n(2n+1) P_{n0}(\cos \psi) = -\frac{1}{4} \operatorname{cosec}^3 \frac{1}{2}\psi - 3 \cos \psi \text{ if } \psi \neq 0. \quad (12)$$

Equations (4) and (6) take into account the fact that data is recorded in relation to the sea surface and not the geoid. Equation (5) can be applied to the instantaneous value of ξ' but Equation (6) has to be evaluated from global stationary fields of ξ' and $\Delta g'$. It is therefore assumed that the effect of tides has been removed from the data prior to use in numerical evaluations. For methods in handling the tides in altimetry data, see (Zetler and Maul 1971; Bretreger 1976; Mather 1977, Sec. 6). In the case of data on land, the value of ξ_s at the element of surface area $d\sigma$ refers to the height of MSL above the geoid at the regional levelling datum.

On considering the shortcomings listed at the commencement of this section, the most favorable procedure for determining ξ_s appears to be the following (Mather, et al., 1976a; Mather 1978). The spectrum of non-tidal quasi-stationary SST (ξ_s) can be considered to be constituted as follows

$$\xi_s = \xi_{ss} + \xi_{s\ell}, \quad (13)$$

where $\xi_{s\ell}$ are components with wavelengths longer than that (ℓ) in the Earth's gravity field which perturb altimeter-satellite orbits above the noise level of the tracking. It is estimated that $\ell = 0 \{10^3 \text{ km}\}$ for 800 km altitudes where the satellite is tracked with ± 10 cm all-weather systems from a global network. ξ_{ss} refers to all contributions with shorter wavelength.

The contributions $\xi_{s\ell}$ to the SST can be determined from the following equations:

EQUATION 1

$$\begin{aligned} \gamma'' &= \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{a}{R}\right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 C_{\alpha nm} S_{\alpha nm}, n \neq 1 \\ &= \gamma\xi' + (W_0 - U_0) - \gamma(\xi_s + k_0\xi_t) - V, \end{aligned} \quad (14)$$

where M is the mass of the Earth with atmosphere, G the gravitational constant and $C_{\alpha nm}$ are harmonic coefficients of degree n and order m . The first equality applies in the space at and exterior to the surface of measurement while the second applies at the surface itself. All coordinates (R, ϕ, λ) are geocentric spherical coordinates.

EQUATION 2

$$\Delta g'' = - \left(\frac{\partial T''}{\partial R} + \frac{2T''}{R} \right) = \frac{GM}{R^2} \sum_{n=0}^{\infty} (n+1) \left(\frac{a}{R} \right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 C_{\alpha n m} S_{\alpha n m}, n \neq 1 \quad (15)$$

$$= \Delta g_d + \frac{2}{R} \left((W_o - U_o) - \gamma \xi_s \right),$$

where

$$\Delta g_d = \Delta g' + \delta g_a = \frac{1}{2} g \xi_d + 2 \frac{T''}{R} c_\phi + o \{ f^2 \Delta g' \}, \quad (16)$$

ξ_d being the deflection of the vertical at the point where gravity (g) is measured and c_ϕ is given by

$$c_\phi = m + f = 3f \sin^2 \phi, \quad (17)$$

where m is defined in Equation (46). The first equality at 15 provides the definition of $\Delta g''$, the second applies in the space exterior to the surface of measurement while the third applies at the latter.

Equations (14) and (15) can be used to determine ξ_s with wavelengths greater than ℓ when the coefficients $C'_{\alpha n m}$ in Equation (9) are known to the equivalent of $o \{ \pm 0.01 - 0.05 \text{ kGal m} \}$. In practice, the satellite determined gravity field model is already incorporated in the higher reference system and hence reflected in the values computed for $\Delta g'$ and ξ' . Thus

$$C_{\alpha n m} = 0 + o \{ 10^{-9} \} \text{ for } n \leq n'. \quad (18)$$

The resulting observation equations take the form

Equation 14 - for values of ξ' from satellite altimetry

$$v_{\xi'} = \left(\xi' + \frac{\delta T''}{\gamma} \right) = \frac{V}{\gamma} = \xi_{sl} = k_o \xi_t + \frac{1}{\gamma} (W_o - U_o), \quad (19)$$

	↓	↓	↓	↓	↓	↓
Order of (m) Magnitude	± 4	± 4 known	$5 \pm \frac{1}{2}$ known	± 2 local value	± 1 local value	± 1 zero degree
Range of (ω) Wavelength	$\omega < \ell$	$0 < \omega < \infty$	$0 < \omega < \infty$	$\omega > \ell$		$\omega = \infty$

and

Equation 15 - for values of $\Delta g'$ in land and continental shelf areas

$$v_{\Delta g'} = \Delta g_d - \frac{2\gamma}{R} (1 - k_o) \zeta_{sd} - \frac{2\gamma}{R} k_o (\zeta_{sl} + \zeta_t) + \frac{2}{R} (W_o - U_o). \quad (20)$$

	↓	↓	↓	↓	↓	↓
Order of (mGal) Magnitude	±10	±10 known	±½ unknown constant	±½ local value	±¼ local value	±¼ zero degree
Range of (ω) Wavelength	ω < 2	0 < ω < ∞	ω = ∞	ω > 2		ω = ∞

The basis for the recovery of the SST by the use of these equations under conditions of unfavorable signal-to-noise is the band-limited nature of the signal being recovered, as can be seen from the wavelength ranges listed above. Equation (20) also constitutes a basis for recovery of the MSL height at the regional elevation datum (ζ_{sd}) as discussed in Section 4.

4. CONDITIONS INFLUENCING DETERMINATIONS OF THE HEIGHTS OF MSL AT COASTAL SITES

The numerical value of ζ_{sd} depends on the basis adopted for the definition of the geoid. Two possibilities exist (Mather 1977):

a. Adopt an oceanic definition for the geoid

The geoid in such a case, is the level surface corresponding to MSL in ocean areas, such a definition can be realized by representing the SST by the relation

$$\zeta_s = k_o(\phi, \lambda) \sum_{n=1}^{n'} \sum_{m=0}^n \sum_{\alpha=1}^2 \zeta_{s\alpha nm} S_{\alpha nm} + \zeta_{ss}, \quad (21)$$

on using Equations (5) and (13), $S_{\alpha nm}$ being given by Equation (10) while $\zeta_{s\alpha nm}$ are surface spherical harmonic coefficients of degree n and order m , n' having the same significance as in Equation (9). For a more comprehensive discussion of this problem, see (Mather, et al., 1978, Sec. 6). On defining ζ'_d by the relation

$$\zeta'_d = \zeta' + (\delta T'' - V)/\gamma \quad (22)$$

observation equations of the form

$$v_{\zeta'} = \zeta'_d - k_o(\phi, \lambda) \sum_{n=1}^{n'} \zeta_{s\alpha nm} S_{\alpha nm} - \left(1 - k_o(\phi, \lambda)\right) \zeta_{sd} + \frac{1}{\gamma} (W_o - U_o) \quad (23)$$

are set up, with the spherical harmonic series only representing ζ_s in oceanic areas. In land areas, $k_o = 0$ and the effect of SST is represented by the value ζ_{sd} at the regional levelling datum.

The observation equation for gravity anomalies takes the form

$$v_{\Delta g'} = \Delta g_d - \frac{2\gamma}{R} k_o(\phi, \lambda) \sum_{n=1}^{n'} \zeta_{s\alpha nm} S_{\alpha nm} - \frac{2\gamma}{R} \left(1 - k_o(\phi, \lambda)\right) \zeta_{sd} + \frac{2}{R} (W_o - U_o) \quad (24)$$

on using Equation (21) in Equation (20). The value of $W_o - U_o$ can be considered to be a known quantity. Its magnitude is dependent on the definition adopted for the geoid, as discussed in Sec. 6.1.

Under such conditions, and as the spherical harmonic modelling and the discrete values are mutually exclusive in Equations (23) and (24), it can be argued that ζ_{sd} can be determined *solely from the analysis of data related to the regional levelling datum*.

b. *Adopt a boundary value problem definition for the geoid*

In this case, ζ_s is defined by the equation

$$\zeta_s = \sum_{n=1}^{n'} \sum_{m=0}^n \sum_{\alpha=1}^2 \zeta_{s\alpha nm} S_{\alpha nm} + \zeta_{ss} \quad (25)$$

instead of Equation (21). The value of ζ generated on land from such a model should be constant for locations on the same regional levelling datum, equal to the ζ_{sd} value for the datum. It would not be unreasonable to expect aliasing effects in coastal areas (*ibid.*) and the portents are not favorable for obtaining estimates of ζ_{sd} for datums with extents smaller than 9^2 km^2 .

It can be concluded that the adoption of a specific definition for the geoid fixes the value of the zero degree term ($W_o - U_o$). This can only be obtained from the global analysis of values of ζ' (and $\Delta g'$, if the second definition is adopted for the geoid). It also follows that the analysis of gravity anomalies on regional geodetic datums can provide a basis for the determination of the value of ζ_{sd}

at the regional levelling datum if the area covered by the datum is larger in extent (10^2 km^2) than the highest full harmonic in the higher reference model which is free from error. The only role played by the satellite altimetry is in defining ($W_0 - U_0$).

In using Equation (24) for land areas ($k_0 = 0$), the density of gravity data points in well represented areas is 1 per 10^2 km^2 while the number of high precision position fixes at which Equation (23) can be used is more likely to be 1 every 10^6 km^2 . The extent of high frequency noise in the more probable gravity anomaly observation equations can be reduced by forming the latter using larger area means, say two degree area means. As the values of such area means are strongly correlated with position (e.g., Mather 1975, p. 77; Mather, et al., 1976b, p. 78) and in view of the adverse signal-to-noise illustrated in Equation (20), it is prudent to model these variations in $v_{\Delta g'}$, which are two orders of magnitude larger than the contribution of the term containing ζ_{sd} . Any two dimensional model should suffice for the task, assuming that the gravity data is evenly distributed about the datum. Thus Equation (24) can be written for land areas on the same levelling datum in the form

$$v_{\Delta g'} = \Delta g_d - \frac{2\gamma}{R} \zeta_{sd} + \sum_n \sum_m \sum_\alpha C''_{\alpha nm} F_{\alpha nm}(\phi, \lambda) + \frac{2}{R} (W_0 - U_0), \quad (26)$$

where $C''_{\alpha nm}$ are harmonic coefficients of the Fourier functions $F_{\alpha nm}$, defined by

$$F_{1nm} = \cos(n \Delta\phi + m \Delta\lambda); F_{2nm} = \sin(n \Delta\phi + m \Delta\lambda), \quad (27)$$

both n and m not being equal to zero simultaneously. $\Delta\phi$, $\Delta\lambda$ in Equation (27) are differences of geocentric surface coordinates from some convenient point of reference in the region. The most important wavelengths which need to be modelled in order that the resulting value of ζ_{sd} is not aliased, are the following:

- a. Those equal to 4 times the smallest dimension (d) of the region served by the datum arising from errors in the assumption described by Equation (18);
- b. Those equal to twice d , due to residual errors in the gravity and levelling networks; and
- c. The error of assumption at 18.

The quality of the determination will depend on

- the extent of the area served by the levelling datum, represented in the solution; and
- whether all wavelengths shorter than λ have been sampled in the determination.

Another factor influencing the determination is the precision with which the regional gravity datum has been established. The required precision for a ± 10 cm determination is $\pm 30 \mu\text{Gal}$ - a precision which can be achieved with modern transportable absolute gravimeters.

It also follows that the quality of the determination will diminish as a function of the shortfall below $\lambda^2 \text{ km}^2$ of the area served by the regional levelling datum. The use of Equation (24) cannot be expected to give stable results if the area sampled is less than $\frac{1}{4}\lambda^2 \text{ km}^2$, even if the modification at Equation (26) were used. In such cases, it becomes necessary to resort to Equation (2) for extrapolating ζ_s from oceanic determinations using satellite altimetry as discussed in Section 2.

A determination of ζ_s on a global basis using Equations (19) and (20) will only define the contributions $\zeta_{s\lambda}$ in Equation (13). There is no reason to believe that the magnitude of ζ_{ss} is any smaller than that of $\zeta_{s\lambda}$. A preferred procedure for establishing ζ_s at tide gauges serving as levelling datums for regions smaller than $\frac{1}{4}\lambda^2 \text{ km}^2$ or at those not connected to levelling networks is the following:

Stage 1

Define $\zeta_{s\lambda}$ at a set of locations about 20 km offshore using Equations (19) and (20).

Stage 2

Having accomplished Stage 1, define ζ_{ss} using Equation (6). 80% of the contribution made by the surface integral is expected from regions in the range $0; 1 < \psi < 5^\circ$ if the higher reference model is used (Coleman and Mather 1976). The balance 20% comes largely from the innermost zone. As $(N_c'' - N_{cp}'')$ in Equation (6) can take both positive and negative values in this region, the possibility exists that the band-limiting constraints placed by the finite footprint of the altimeter, results in the loss of high frequency signal. The significance of this depends on the amplitude of very short wavelength contributions (< 20 km) to ζ_s . This effect can be disregarded if less than ± 5 cm, noting that the quantity required is the annual average SST.

Stage 3

Use current meter data, atmospheric data and models of frictional forces to extrapolate ζ_s obtained at oceanic sites within 20 km of the coastal site in Stages 1 and 2, to the latter using Equation (2). The resulting value should agree with that obtained from processing all gravity anomaly and height anomaly values related to the datum, provided adequate coverage existed for the purpose.

Solutions for ζ_{ss} in Stage 2 are also influenced by the occurrence of the term ζ_s within a surface integral, as discussed in (Mather 1977, Sec. 7). It is therefore necessary to solve for ζ_{ss} as a regional field using models of the type described in Equations (26) and (27). This, in turn, requires that $\Delta g'$ be defined as a regional field in this basic oceanic area with a precision of ± 0.03 mGal through wavelengths of interest. In theory, the minimum wavelength in ζ_{ss} which can be recovered, is governed by the Nyquist limit which is a function of the satellite altimeter footprint. In practice, it is desirable to reduce the frequency range in ζ_{ss} in view of the difficulty of defining $\Delta g'$ to the required precision, free from the effects of variations in ζ' with periods shorter than that implied in the quasi-stationary concept (i. e., less than a year). It would be most helpful if it were established that quasi-stationary SST over this period had a power spectrum to which the contributions of wavelengths below some lower limit (say, 10^2 km) were less than 20 cm^2 .

Another problem in the evaluation of ζ_{ss} at Stage 2 is the necessity to define N_c'' (Equation (11)) as a continuous field within 500 km of the point of computation. This would call for the determination of ζ' to ± 10 cm in all *land areas* which fall within this region. The only means of achieving this objective at the present time, is by collocating transportable laser tracking systems at points in the first order geodetic levelling network in the area. Ideally, ζ' (and hence N_c'') should be defined at points on a 10^2 km grid. The cost of such an operation is prohibitive.

5. PRACTICAL CONSIDERATIONS

It has been shown in Sections 3 and 4 that satellite altimetry has the potential to:

- a. select a particular level surface of the Earth's gravity field as the geoid at the ± 1 -5 cm level by defining a magnitude for $(W_0 - U_0)$; and
- b. define the height of MSL above the geoid so selected at coastal sites in conjunction with gravity, oceanographic and geodetic surveys.

The precision requirements to be met by the various types of data are the following:

a. *Satellite altimetry data*

Sea surface heights ζ' must be computed from orbital ephemerides with a resolution of ± 10 cm in the radial component of position.

b. *Global gravity field model to degree n'*

The global gravity field model consistent with the above orbital ephemerides to the noise level of the tracking should be such that when used to define the higher reference model, the resulting values of T'' have no terms with wavelengths greater than ℓ (equivalent to n'), ℓ being estimated to be $\approx 10^3$ km.

c. *Regional levelling surveys within 500 km of the computational area*

The precision of such control surveys used in the control of gravity anomaly and height anomaly computations, should cover continental extents.

d. *Regional gravity control networks*

The precision required on land is $\pm 30 \mu\text{Gal}$ in all control networks with wavelengths greater than ℓ .

e. *Oceanographic surveys for extrapolation to coastal sites*

The precision required for current velocity measurements is 1 cm s^{-1} and for frictional forces $10^{-4} \text{ cm s}^{-2}$ if a ± 1 cm resolution is to be obtained in the extrapolation of ζ_s . The values required are the average for the sampling period.

The desired precisions are not currently available for data at (a), (b), and (d). In the case of data at (c), the favorable indications from the internal statistics of level net adjustments need to be tempered by the doubts implied in estimates of coastal SST especially in the meridional direction, as obtained from levelling/tide gauge comparisons.

An attempt to improve the gravity data bank for Australia for use in SST determinations resulted in a gravity anomaly representation which was assessed as being sufficient only for determinations to ± 30 cm in ζ' . (Mather, et al., 1976b). None of the other data banks are likely to be of better quality.

The quality of global gravity field models currently available, has been the subject of close scrutiny, especially with the advent of satellite altimetry data of quality from the GEOS-3 mission. Recent studies of GEOS-3 altimeter data off eastern Australia show discrepancies between geoidal and sea surface models which appear to have amplitudes of up to 5 m and wavelengths of 2000 km (Mather, et al., 1977, p. 36). On the other hand, it is possible to manipulate GEOS-3 altimetry data so that the dominant features of the global quasi-stationary sea surface topography are recovered with a precision estimated at ± 6 cm (Mather, et al., 1978, Sec. 9). It was therefore decided to use the techniques described above to:

- define a geoid consistent with the 1977 GEOS-3 altimeter data bank; and
- establish, if possible, the height of the datum level surface implicit in the current gravity data banks for Australia and the United States.

While some doubt exists about the practical significance of the latter results, the computations would highlight the nature of the numerical problems encountered in the evaluation. These computations are described in Section 6 and the results discussed in Section 7.

6. NUMERICAL RESULTS FROM GEOS-3 DATA

6.1 The Geoid for Epoch 1976.0

See Section 4. As abnormal conditions may prevail in coastal areas, it is preferable to select the datum level surface on the basis of data sampled in ocean areas alone.

The Data Set

The GEOS-3 altimetry used in the definition of the geoid for epoch 1976.0 were the total data available in the 1977 data bank at Goddard for the periods 1 September to 31 October 1975 and 1 March to 30 April 1976 (the Equinox Data Set for 1976.0). 350 passes of data were recorded during the first period and 284 during the second. The total distribution of data is shown in Figure 1. This data was used to obtain a geometrical model of the sea surface consistent with the best available orbits as described in (Mather, et al., 1978, Sec. 5). The resulting model is based on data which is minimally affected by the seasons. The representation obtained, however, is less than desirable due to the irregular data acquisition.

The Computational Procedure

The instantaneous position X_i of the sea surface at epoch ($\tau = t$) is obtained from the Earth space satellite coordinates $X_{is}(t)$ on a three dimensional Cartesian coordinate system X_i related to the geocenter, the CIO pole and the meridian of zero longitude (Greenwich), and the gravitationally stabilized altimeter range $h(t)$ to the sea surface, by the relation

$$X_i(t) = X_{is}(t) - h(t)l_i(t) + o\{10^{-8}h\}, \quad (28)$$

where $l_i(t)$ are the direction cosines of the normal which passes through the satellite position. The coordinates X_i are easily converted to geocentric spherical coordinates (R_o, ϕ, λ) by the well known relations

$$R_o = \left(\sum_{i=1}^3 X_i^2 \right)^{1/2}; \lambda = \tan^{-1}(X_2/X_1); \phi = \tan^{-1}[X_3/(X_1^2 + X_2^2)^{1/2}]. \quad (29)$$

If the coefficients of the Earth's gravity field model are "free from error" to degree n' , the geopotential at the sea surface W_{ss} can be modelled using the harmonic coefficients $C'_{\alpha nm}$ ($n \leq n'$) using the relation

$$W_{ss} = \frac{GM}{R_o} \sum_{n=0}^{n'} \left(\frac{a}{R_o} \right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 C'_{\alpha nm} S_{\alpha nm} + \frac{1}{2} R_o^2 \cos^2 \phi \omega^2, \quad (30)$$

where $S_{\alpha nm}$ are defined by Equation (10), and a is the Earth's equatorial radius. The model for the potential V_s of the atmosphere exterior to it is given by

$$V_s = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R_a}{R} \right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 V_{s\alpha nm} S_{\alpha nm}, \quad (31)$$

where R_a is the radius of the minimum geocentric sphere enclosing the Earth's atmosphere. Downward continuation through the atmosphere is only possible when considering the potential W_e of the solid Earth and oceans, given by

$$W_e = W - V = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{a}{R} \right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 C''_{\alpha nm} S_{\alpha nm}, \quad (32)$$

where

$$C''_{\alpha nm} = C'_{\alpha nm} - \left(\frac{R_a}{a} \right)^n V_{s\alpha nm}. \quad (33)$$

The geopotential W_{ss} at the sea surface is given by

$$W_{ss} = (W_e)_{ss} + V_g, \quad (34)$$

where V_g is the potential of the atmosphere as evaluated at the sea surface. As the atmospheric potential is dominated by the term of zero degree (Anderson, et al., 1975, p. 33) and also contains harmonics of degree 1, considerable computer economy is achieved by representing V_g by a surface harmonic model

$$V_g = \sum_{n=0}^{\infty} \sum_{m=0}^n \sum_{\alpha=1}^2 V_{g\alpha nm} S_{\alpha nm} + o\{0.02 \text{ kGal m}\}. \quad (35)$$

Thus the low degree harmonic estimate ($n \leq n'$) W_{ss} of the geopotential W at the sea surface can be written as

$$W_{ss} = \frac{GM}{R_o} \sum_{n=0}^{\infty} \left(\frac{n}{R_o}\right)^n \sum_{m=0}^n \sum_{\alpha=1}^2 C'_{\alpha nm} + S_{\alpha nm} + \frac{1}{2} R_o^2 \cos^2 \phi \omega^2, \quad (36)$$

as evaluated at (R_o, ϕ, λ) defined by Equation (29), where

$$C_{\alpha nm} = C'_{\alpha nm} + \delta C_{\alpha nm}. \quad (37)$$

The coefficients $C'_{\alpha nm}$ in Equation (37) are satellite-determined harmonic coefficients of the type embodied in the model GEM 9 (Lerch, et al., 1977), while $\delta C_{\alpha nm}$ are the corrections required in downward continuing the satellite determined geopotential for determining W_{ss} at the sea surface, being defined by the relations

$$\delta C_{\alpha nm} = \frac{R_o}{GM} V_{g\alpha nm} - \left(\frac{R_g}{a}\right)^n V_{s\alpha nm} + o\{\delta C_{\alpha nm}\}. \quad (38)$$

Table 1 sets out the corrections $\delta C_{\alpha nm}$ computed from Anderson's evaluation of the atmospheric potential at both satellite altitudes and the Earth's surface (Anderson 1976). The effect of the corrections to W_{ss} per coefficient never exceed ± 5 kGal mm for degrees up to (5,5). This is due to the gravitational effect of the long wave components (> 2000 km) contribute 98 percent of the strength of signal which vary less than ± 5 kGal cm over the surface of the oceans (ibid., pp. 209-210). Consequently, the differential effect is insignificant through low degree terms.

Under these circumstances, W_{ss} can be computed from the coefficients C'_{ann} instead of C_{ann} in Equation (36).

Evaluation of W_0

The 634 passes of GEOS-3 altimetry between latitudes -65°N and 65°N recorded in September-October 1975 and March-April 1976 provided a 39.8 percent representation of the 33,902 equi-angular $1^\circ \times 1^\circ$ squares classified as oceanic in this study. The rms residual representing variations within a square was $\pm 4.4\text{m}$. The resulting value of W_0 obtained was

$$W_0 = 6,263,682.76\text{kGal m}, \quad (39)$$

based on the Bermuda calibration of the GEOS-3 altimeter, and

$$GM = 3.986\,004\,7 \times 10^{20} \text{cm}^3 \text{s}^{-2}, \quad (40)$$

consistent with the velocity of light c taking the value

$$c = 2.997\,924\,58 \times 10^{10} \text{cm s}^{-1}. \quad (41)$$

This value has an estimated uncertainty of $\pm 0.4\text{m}$. For details on how the data was processed, see (Mather, et al., 1978, Sec. 3). (A summary of results is given in Table 4.)

6.2 The Reference System Used

The system of reference is defined by the gravity field model GEM 9 whose coefficient

$$C_{20} = -1.082\,627\,(6) \times 10^{-3} \quad (42)$$

is consistent with the value of c at (41). On adopting

$$a = 6,378,140.00\text{m}, \quad (43)$$

it follows that the potential U_0 on the surface of the rotating equipotential ellipsoid of reference is (e.g., Mather 1971, p. 83)

$$U_0 = \frac{GM}{a} \frac{\alpha}{\sin \alpha} + \frac{1}{3} a^2 \omega^2, \quad (44)$$

being related to C_{20} by the relation (o.g., Mather 1978, App.)

$$C_{20} = -\frac{\sin^2 \alpha}{3} \left[1 - \frac{2}{15} \frac{m \sin \alpha}{q_2(\alpha)} \right], \quad (45)$$

where $\alpha = \cos^{-1} [1 - f]$,

$$m = a^3 \omega^2 / GM, \quad (46)$$

and $q_2(\alpha)$ is given by

$$q_2(\alpha) = \frac{1}{2} [\alpha (3 \cot^2 \alpha + 1) - 3 \cot \alpha]. \quad (47)$$

The reference quoted sets out simplified procedures for solving Equation (45). U_0 for the system of reference adopted in the present series of calculations is

$$U_0 = 6,263,682.67(6) \text{ kGal m}. \quad (48)$$

6.3 The Computation of the Gravity Anomaly

The correct procedure for preparing gravity data for high precision computations is described in (Mather, et al., 1976b). The data maintained in most gravity data banks (e.g., Rapp 1977) is in the form of free air anomalies Δg_f , on some system of reference, usually Geodetic Reference System 1967 (GRS 67), computed from the formula

$$\Delta g_f = g - \gamma_0 + 0.3086 h(m) \quad (49)$$

where γ_0 is normal gravity computed for the equipotential ellipsoidal model using the formula

$$\gamma_0 = \gamma_e (1 + \beta \sin^2 \phi_g + \beta_2 \sin^2 2\phi_g) + o \left\{ 0.05 \text{ mGal} \right\}, \quad (50)$$

where

$$\gamma_e = 978,031.675 \text{ mGal} \quad (51)$$

$$\beta = 5.30254 \times 10^{-3}; \beta_2 = -5.862 \times 10^{-6}.$$

As pointed out in (ibid.), resolution to $\pm 4 \mu\text{Gal}$ can be obtained by using the formula

$$\gamma_0 = \gamma'_e(1 + \beta'_1 \sin^2 \phi_g + \beta'_2 \sin^4 \phi_g), \quad (52)$$

where

$$\begin{aligned} \gamma'_e &= 978,031.678 \text{ mGal} \\ \beta'_1 &= 5.27893 \times 10^{-3}; \beta'_2 = 2.346 \times 10^{-6}, \end{aligned} \quad (53)$$

The gravity anomaly Δg has to be re-computed using the formula

$$\Delta g = \Delta g_f - 0.3086 h(m) - \frac{2\Delta W}{a} \left(1 + c_\phi + \frac{\Delta W}{2a\gamma} \right), \quad (54)$$

where c_ϕ is defined by Equation (17) and ΔW is related to increments of geodetic levelling dz by the relation

$$\Delta W = - \int_{\text{MSL Datum}}^P g dr. \quad (55)$$

For a description of these calculations for the Australian gravity data bank, see (ibid., p. 58). The gravity data for the United States was in the form of free air anomalies originally computed by the Defense Mapping Agency Aerospace Center (DMAAC). The normal gravity was computed using closed formulae (e.g., Mather 1971, p. 88). Conversion to gravity anomalies was obtained by using Equation (55) on the gravity and elevation data banks for the Central North America and using the resulting set of geopotential differences related to the Galveston tide gauge.

All gravity anomalies were finally referred to the higher reference model defined by GEM 9 and the constants defined by Equations (41) and (43).

7. COMPUTATION OF DATUM LEVEL SURFACE DISPLACEMENTS FROM THE GEOS-3 GEOID FOR THE EPOCH 1976.0

7.1 The Jervis Bay Datum Level Surface

All Australian gravity data is related to a freely adjusted Australian Levelling Survey of 1970 and referred to the Jervis Bay Datum at ($\phi = 35.1^\circ\text{S}$, $\lambda = 150.7^\circ\text{E}$)

as described in (Mather, et al., 1976b, Sec. 2). In a first stage, the gravity anomalies Δg were converted to the pseudo-anomalies Δg_d using Equation (16). In preferred circumstances, the higher reference model used should be free from error. Assuming that ξ_{sd} is required to ± 10 cm, it follows that Equation (18) must also hold for all coefficients included in the higher reference model, assuming the number of coefficients to be about 400. A solution procedure based on Equation (26) will be subject to considerable aliasing of the value of ξ_{sd} if the errors in the higher reference model with wavelengths greater than the shortest dimension (d) of the area served by the datum, were not modelled in the computations. It is estimated that the error in the GEM 9 coefficients to (4, 4) on models at the surface of the Earth is $\pm 1.4 \times 10^{-8}$ (Lerch, et al., 1977, p. 52), equivalent to approximately ± 9 kGal cm in T'' . These estimated errors increase rapidly with increase of n to around ± 60 kGal cm for degree 20.

The value of ξ_{sd} can in principle, be obtained by the analysis of either the $1^\circ \times 1^\circ$, $2^\circ \times 2^\circ$ or $5^\circ \times 5^\circ$ data banks. The results obtained are influenced by the following factors:

- The signal-to-noise. ξ_{sd} is not larger than ± 2 m while the variability of the data increases with decrease of square size (Table 2, Row 3).
- Departures from the assumption that the gravity field model is error free. The existence of a large non-zero value for the regional mean (Δg_d) of Δg_d over Australia emphasizes the need for Fourier modelling the long wavelengths errors in the gravity field. The large positive values of Δg_d for Australia (Table 2, Row 2) indicate the net high of surface gravity in the region. These values are highly correlated with position showing net highs in the east and west of the continent with a band of lows in the center (e.g., Mather, et al., 1976b, p. 78; Lerch, et al., 1977, p. 71). This type of effect has a wavelength two-thirds that of the east-west dimension of the continent and should be modelled when using Equation (26).
- Errors in the area means. These arise primarily due to inadequate sampling.

It was therefore decided to model the following wavelengths in the Fourier series when effecting a solution:

$$\frac{2}{3} d, \frac{4}{3} d, 2d, \frac{8}{3} d, \dots$$

The values adopted for d in the Australian calculations were $d_\phi = 30^\circ$ in latitude and $d_\lambda = 45^\circ$ in longitude.

In view of the unfavorable signal-to-noise, it was necessary to constrain the solution to an a priori assessment of the magnitudes of the corrections. For example, the term $\gamma S_{sd}/R$ in Equation (26) will not exceed ± 0.3 mGal while the coefficients $C''_{\alpha nm}$ should on the average, not be significantly larger than $\overline{\Delta g_d}/N$, where N is the total number of harmonics modelled. Consequently, the solutions shown in Tables 2 to 4 were obtained by minimizing

$$\Phi = \sum_{i=1}^{N'} w_i \Delta g_i^2 + \sum_{\alpha=1}^N w_{c\alpha} (C''_{\alpha nm})^2, \quad (56)$$

where

$$w_i = 1/\cos \phi_i; \quad w_{c\alpha} = \begin{cases} 100 & \text{if } \alpha = 1, n = m = 0 \\ (N/(\overline{\Delta g_d}))^2 & \end{cases} \quad (57)$$

The solutions obtained for Australia using AUSGAD 76, GEM 9 and the GEOS-3 altimeter-determined geoid for 1976.0 are set out in Tables 2 and 5. The preferred result is obtained using fully represented $5^\circ \times 5^\circ$ area means as the area means are probably more reliable, being less affected by irregularities in gravity field sampling. The number of observation equations is limited, reducing to 15 if only fully represented squares (i.e., only $5^\circ \times 5^\circ$ squares based on $25^\circ \times 25^\circ$ values) were considered (Table 5).

On this basis, the preferred value for the height of MSL at the Jervis Bay Datum is

$$(\zeta_{sd})_{\text{Jervis Bay}} = +0.21 \text{ m}. \quad (58)$$

The equivalent value as extrapolated from the deep oceans using oceanographic data is $+0.1 \pm 0.2$ m (Mather, et al., 1978, Figure 1), noting that a zero degree effect of $+1.14$ m has been eliminated. The figure at (58) is referred to epoch 1968.0. The variation of the height of MSL with time at Sydney is estimated at $+1$ mm per year. Thus there is less than 1 cm discrepancy introduced into the result due to the non-coincidence of epochs of the levelling and the altimetry. The error in the datum for the Australian gravity is estimated at ± 0.06 mGal (Mather, et al., 1976b, p. 79), introducing an uncertainty of 0.18 m in the result at (58). For estimates of other sources of error, see comments on the result at (59).

7.2 Estimating the Effects of Zero Degree in the Gravity Data Bank for Central North America

The region covered by this study was the North American continent bounded by the parallels 28°N and 50°N. This included a small part of Mexico and the south-eastern part of Canada. Gravity values on the North American continent are, as best as possible, referred to the International Gravity Standardization Network (IGSN 71) (Morelli, et al., 1971). The basic network was assembled by the Defense Mapping Agency Aerospace Center. It would be difficult to assess, without a major re-examination of the data, whether the pattern of errors in the United States Levelling Network are reflected in the resulting 1° x 1° free air anomaly data bank compiled by Rapp (1977). The data used in this study had been rounded off to the nearest mGal. Its characteristics are summarized in Table 3. Parts of the Canadian gravity data bank were also included in this study. The same comments made about the elevations of gravity stations in the United States apply to those in Canada, there being a variable systematic difference between common junction points of the two levelling systems which is about +10 cm on the average (Lachapelle 1978). This has not been considered significant in the present study which is of an exploratory nature.

It is therefore not clear that the analysis of the gravity anomaly data bank for central North America, prepared as described in Section 6.3, will contain any information on the height of MSL at the datum level surface for the region, as implied in the computation of free air anomalies. As a starting point, it was decided to adopt the suggestion that tide gauge at Galveston be adopted as a suitable datum for this levelling network (Holdahl 1978). Geopotential differences were computed using 1° x 1° mean square elevation and gravity data banks in relation to the value in the 1° x 1° square ($\phi = 29.5^\circ\text{N}$; $\lambda = 261.5^\circ\text{E}$) containing this site.

These data banks and the resulting geopotential network were used in Equation (54) to produce a gravity anomaly data bank for central North America as defined above. The characteristics of the data used in the analysis are shown in Table 3. As mentioned in the previous section, the discrepancy between the GEM 9 model and the surface gravity data, as embodied in the value of $\overline{\Delta g_d}$ for the region is five times smaller than that for Australia (Tables 2 and 3, Row 2). This is probably a reflection of the better tracking coverage available in the North American area when compiling the GEM 9 model.

If it were assumed that all the gravity data in the North American study were

- based on a regional standardization network of the same quality as IGSN 71; and

- converted to gravity anomalies based on a network of elevations substantially controlled by the freely adjusted regional levelling network,

it can be said that the height of MSL at Galveston is given by

$$(\zeta_{sd})_{\text{Galveston}} = +0.14 \text{ m} . \quad (59)$$

It is only possible to obtain a very approximate oceanographical value for ζ_{sd} at Galveston as $+0.1 \pm 0.3 \text{ m}$ (Levitus and Dort 1977, p. 1283), allowing for the zero degree effect. The sources of uncertainty in the result at (59), provided the above assumptions were valid, are the following:

- $\pm 20 \text{ cm}$ due to errors in the gravity standardization network. This figure is a guess, compatible with the more carefully assessed figure for the Australian national network, quoted in Section 7.1.
- $\pm 12 \text{ cm}$ due to aliasing as a result of using too few coefficients in the Fourier modelling - an inevitable consequence when using area means for improving the signal-to-noise.
- The value of W_0 obtained in Section 6.1 was not based on a full coverage of the oceans between 65°S and 65°N . As shown in Table 4, the result may require revision by up to $\pm 30 \text{ cm}$ as further orbital refinements are made.

It is not unreasonable to conclude that the values of ζ_{sd} given in Equations (58) and (59) have uncertainties at the $\pm 0.4 \text{ m}$ level. The level of agreement obtained with oceanographic values is much better, being about one-fourth this value (i.e., $\pm 0.1 \text{ m}$).

8. DISCUSSION OF RESULTS

The results presented in Section 7.1 and 7.2 are based on the following data:

- A geoid for epoch 1976.0 based on data in the 1977 GEOS-3 altimeter data bank. This data base is being added to and in the process of further revision. It is not expected that the value of W_0 given in Section 6.1 will change by more than $\pm 0.3 \text{ kGal m}$ when the representation increases from the 39.6 percent coverage used in the present study and when refinement is complete.

- The gravity anomaly data bank for Australia specially prepared for sea surface topography determinations (AUSGAD 76).
- The $1^\circ \times 1^\circ$ free air anomaly data set for central North America originally compiled by the DMA Aerospace Center and provided by Rapp in the form of values rounded off to the nearest mGal.
- The GEM 9 gravity field model.

The last named data set is not critically involved in the determination though weaknesses in the model cause additional signal-to-noise problems.

The following observations can be made on the results presented in Sections 6 and 7:

- a. The analysis of the data for Australia (Tables 2 and 5) indicate the extent of the aliasing influence of $5^\circ \times 5^\circ$ area means which were not based on a full representation of surface gravity data (i. e., twenty five $1^\circ \times 1^\circ$ values). Restriction of the analysis to fully represented areas reduces the ratio of unknowns to observation equations. This is offset by the reduction of noise in the observational data and results in an improved solution. Stability of solution is enhanced by restricting the Fourier modelling to the same range of longitude per parallel sampled.
- b. The results for Australia indicate that the use of this technique in regions not providing heavy tracking coverage for the development of the satellite determined gravity field model, will produce conditions where k_{sd} has to be determined in the presence of adverse levels of noise. Subsequent computational instability can be avoided by studying the nature of the distribution of Δg_d over the region before the selection of wavelengths for Fourier modelling.
- c. The results given in this paper for the MSL datum at Galveston are based on the assumption that the gravity anomaly data bank for central North America was based on the geodetic levelling. There is no assurance that this is the case. It is most desirable that this experiment be repeated with a gravity data set whose elevations are known to be related to the continental levelling network based on the Galveston Datum Level Surface.
- d. The results presented in this analysis establish the potential of this method for defining the height of MSL at the regional levelling datum serving areas larger than the square of the minimum wavelength in the satellite determined gravity field model. Ideally, the model should

be free from error through these wavelengths. However, slightly degraded results can be obtained even if this condition is not satisfied, as seen from the results given above.

- e. This study shows that gravity anomalies computed from levelling data related to either the Jervis Bay or Galveston Datums can be assumed to refer to the geoid to 0.1 mGal.
- f. This technique also provides an exacting test of the value of GM used in Equation 40.

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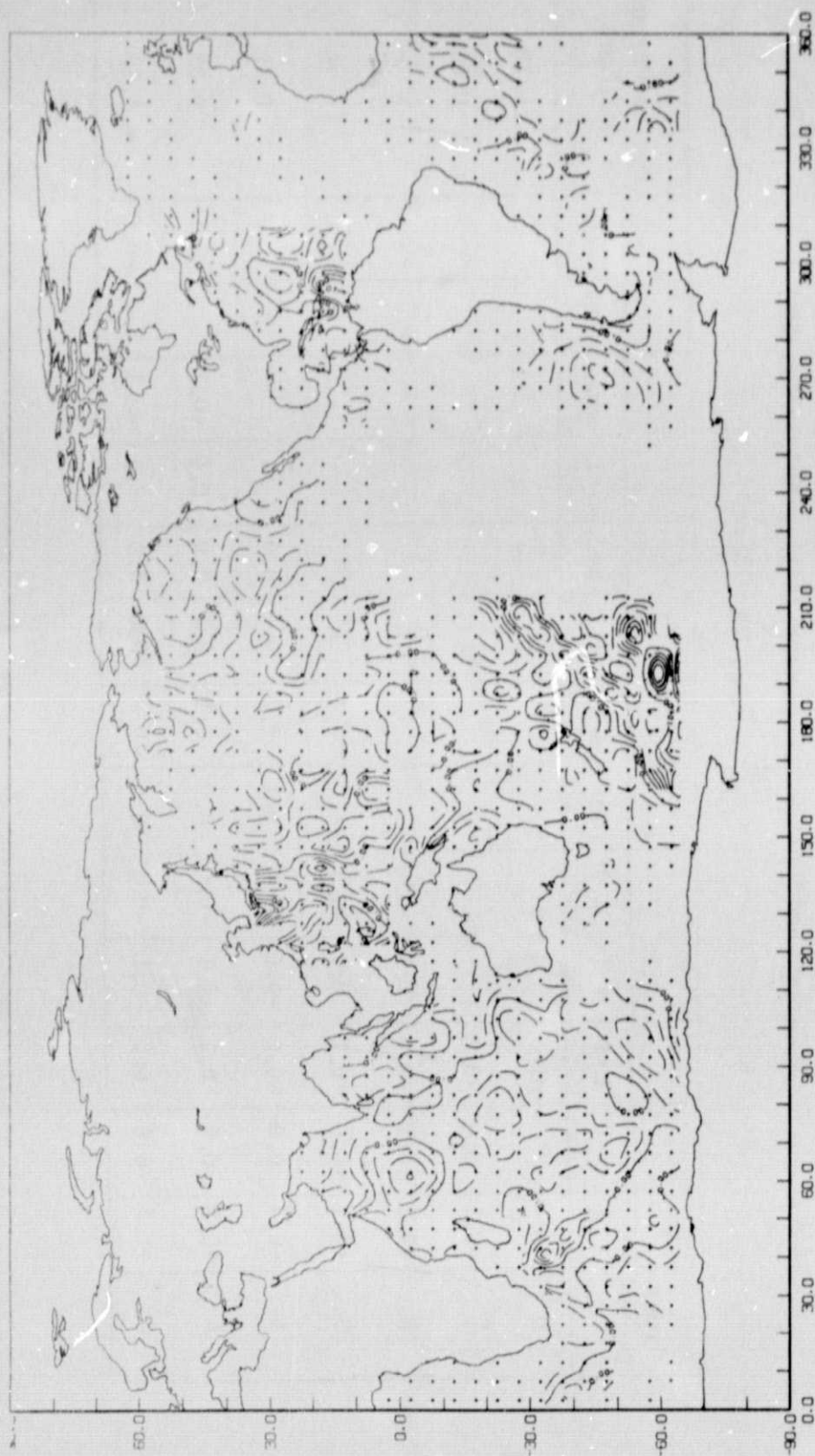


Figure 1. Distribution of 5° x 5° Area Mean Sea Surface Heights Used to Compute W_0
 (Contours Represent $|W_{ss} - W_0|$ in kGal m from Raw Wallops Orbits)
 Contour Interval 2 kGal m

Table 1
The Differential Effect of the Atmosphere in Geopotential Computations from Satellite
Determined Potential Coefficients (δC_{anm} in Equation (38))
 $-\frac{GM}{R} \delta C_{anm}$ (in kGal cm)

Order Degree	0		1		2		3		4		5		6		Degree Variance (kGal cm) ²
	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	
0	-0.4														0.19
1	+0.1	0.0	-0.2												0.05
2	+0.2	-0.0	-0.2	0.0	-0.0										0.05
3	+0.2	+0.0	+0.1	0.1	-0.5	+0.0	+0.1								0.26
4	-0.4	0.0	-0.0	-0.0	+0.0	0.1	0.2	0.0	0.0						0.21
5	+0.2	0.0	0.1	-0.1	-0.0	0.0	0.1	0.1	0.0	0.0	-0.1				0.08
6	+1.1	-0.0	0.0	0.1	-0.0	-0.0	-0.0	-0.0	-0.0	-0.1	-0.1	-0.1	0.0	-0.0	1.13

Table 2

Statistics From Area Mean Values of Δg_d (Equation (16)) in the Australian Gravity Data Bank (AUSGAD 76), Based on the Freely Adjusted Level Network for Australia Referred to the Jervis Bay Datum Level Surface (Units mGal)

 $d_\phi = 30^\circ$ $d_\lambda = 45^\circ$

Square Size	$1^\circ \times 1^\circ$	$2^\circ \times 2^\circ$	$5^\circ \times 5^\circ$
No. of Blocks	722	181*	30* (15)
Mean Value ($\overline{\Delta g_d}$)	3.65	3.81	3.41 (-0.18)
rms	16.6	13.3	8.3 (6.5)
Expected $ \zeta_{sd} / \overline{\Delta g_d} $	0.02	0.02	0.04 (0.05)

*Minimum Representation = 40 percent

(Figures within brackets for $5^\circ \times 5^\circ$ squares are based on a sample which includes only squares where mean is computed from 25 $1^\circ \times 1^\circ$ values; i.e., 100% representation)

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Table 3

Statistics from Area Mean Values of Δg_d (Equation (16)) in the Gravity Data Set for Central North America, Based on Geopotential Estimates Related to The Galveston Datum Level Surface (Units mGal)

$d\phi = 20^\circ$

$d\lambda = 45^\circ$

Square Size	$1^\circ \times 1^\circ$	$2^\circ \times 2^\circ$	$5^\circ \times 5^\circ$
No. of Blocks	835	218*	34*
Mean Value ($\overline{\Delta g_d}$)	-0.75	-0.67	-0.25
rms	15.9	10.4	6.5
Expected $ \xi_{sd} / \overline{\Delta g_d} $	0.03	0.03	0.05

*Minimum Representation = 40 percent

Table 4

The Potential of the Geoid (W_0) from GEOS-3 Altimetry
 $GM = 3.986\ 004\ 7 \times 10^{20} \text{cm}^3 \text{s}^{-2}$

Data Source	Wallops	Wallops
Epoch	Sep-Oct '75 Mar-Apr '76	Feb-Aug '76
No. of Passes	634	882
No. of 1° Sq. Sampled	13,499	12,349
rms ($W_{ss} - W_0$) \pm kGal m	5.8	5.1
W_0 (kGal m)	6,263,682.76	6,263,682.39

Table 5

Modelling the Discrepancies Between Regional Gravity Data Banks (5° x 5° area means) and GEM 9 (in mGal) [Equations (26), (56) and (57) apply]

Region Representation		Australia*			Central North America**		
		40		100		40	
n	m	1	2	1	2	1	2
$\frac{2}{3}d_\phi$	0	-2.6	-0.0	-0.1	-0.0	-0.0	+0.1
0	$\frac{2}{3}d_\lambda$	-0.0	-3.5	-0.0	-0.0	-0.0	+0.1
$\frac{4}{3}d_\phi$	0	+3.5	-1.9	-0.0	+0.0	-0.1	+0.1
0	$\frac{4}{3}d_\lambda$	+4.3	+2.1	-0.0	+0.0	-0.1	+0.3
$2d_\phi$	0	-1.1	-2.3	+0.0	+0.0	-0.1	+0.0
0	$2d_\lambda$	+2.4	-1.5	+0.1	+0.0	+0.1	+0.2
$\frac{8}{3}d_\phi$	0	+1.4	+4.5	+0.0	+0.0	-0.1	-0.1
0	$\frac{8}{3}d_\lambda$	+2.0	+3.4	+0.0	-0.0	-0.0	-0.2
No. of Data Points		30		15		34	22
$\overline{\Delta g_d}$ (mGal)		+3.4		-0.2		-0.3	-0.9
rms Variation (\pm mGal)		8.3		6.5		+6.6	+6.6

* $d_\phi = 30^\circ$; $d_\lambda = 45^\circ$

** $d_\phi = 20^\circ$; $d_\lambda = 40^\circ$

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16. Abstract <p>Techniques are described for determining the height of Mean Sea Level (MSL) at coastal sites from satellite altimetry. Such information is of value in the adjustment of continental leveling networks. Numerical results are obtained from the 1977 GEOS-3 altimetry data bank at Goddard Space Flight Center using the Bermuda calibration of the altimeter. Estimates are made of the heights of MSL at the leveling datums for Australia and a hypothetical Galveston datum for central North America. The results obtained are in reasonable agreement with oceanographic estimates obtained by extrapolation. It is concluded that all gravity data in the Australian bank AUSGAD 76 and in the Rapp data file for central North America refer to the GEOS-3 altimeter geoid for 1976.0 with uncertainties which do not exceed ± 0.1 mGal.</p>			
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